ROTATIONAL MECHANICS_(Moment of Inertia, Torque, Fixed axis Rotation) PART-A LEVEL-I_(THEORY)

ysics Sir.

implifying learning Physics

SINGLE ANSWER CORRECT

1. A uniform rectangular metal plate ABCD is supported by two hinges P_1 and P_2 so that it remains in equilibrium in vertical plane (with line AB vertical) as shown in the figure. The reaction forces R_1 and R_2 exerted by hinges on the metal plate is best depicted by :



- 2. A ladder is at rest with its upper end against a wall and its lower end on the ground. A worker is about to climb it. When is it more likely to slip?
 - (a) Before the worker is on it.
 - (b) When the worker is on the lowest rung
 - (c) When the worker is halfway up the ladder
 - (d) When the worker is on the top rung
- 3. Newton's second law for translational motion in the xy plane is $\Sigma \vec{F} = m\vec{a}$; Newton's second law for rotation is $\Sigma \tau_z = I\alpha_z$. Consider the case of a particle moving in the xy plane under the influence of a single force.
 - (a) Both $\Sigma \vec{F} = m\vec{a}$ and $\Sigma \tau_z = I\alpha_z$ must be used to analyze the motion of this particle
 - (b) Either $\Sigma \vec{F} = m\vec{a}$ or $\Sigma \tau_z = I\alpha_z$ can be used to analyze the motion of this particle
 - (c) Only $\Sigma \vec{F} = m\vec{a}$ needs to be used to analyze the motion of this particle
 - (d) Only $\Sigma \tau_z = I\alpha_z$ can be used to analyze the motion of this particle

4. Two spheres having masses m_1 and m_2 are kept on an inclined plane as shown. Both of them are in equilibrium. What is a necessary condition for this ? (μ_1 is coefficient of friction between m_1 and inclined plane and μ_2 is coefficient of friction between m_2 and inclined plane):



5. A bead constrained to move on rod in gravity free space as shown in figure. The rod is rotating with angular velocity ω and angular acceleration α about its end. If μ is coefficient of friction. Mark the correct option. Rod rotates in the plane of paper :



A) If $\mu = \frac{\omega^2}{\alpha}$ friction on bead is static in nature

B) If
$$\mu > \frac{\omega^2}{\alpha}$$
 friction on bead is kinetic in nature
C) If $\mu < \frac{\omega^2}{\alpha}$ friction is static

- D) If bead does not slide relative to rod. Friction will not exist between bead and rod
- 6. Two strings support a uniform rod as shown. String at end B is cut. Which of the following is true just after cut ?



[P] initial acceleration of A is vertical

[Q] initial acceleration of A is horizontal

[R] initial acceleration of centre of mass of rod is vertical

[S] initial acceleration of centre of mass of rod is horizontal

A) P and Q B) Q and R C) R and S D) P and S

7. Statement – 1 : A pair of upright metersticks, with their lower ends against a wall, are allowed to fall to the floor. One is bare, and the other has a heavy weight attached to its upper end. The stick to hit the floor first is the weighted stick.

Statement -2: The torque acting on weighted stick is more than the bare stick.

A) Statement -1 is true, statement -2 is true and statement -2 is the correct explanation for statement -1

B) Statement -1 is true, statement -2 is true and statement -2 is not the correct explanation for statement -1

C) Statement -1 is true, statement -2 is false

D) Statement -1 is false, statement -2 is true

ONE OR MORE THAN ONE ANSWER CORRECT

8. A rigid body rotating with some non-zero angular velocity $(\overline{\omega_0})$ and non-zero angular acceleration $(\overline{\alpha_0})$ is viewed in a mirror. The axis of rotation is parallel to the mirror. Consider the motion of a point 'P' on the rigid body. The position vector of P is $\overrightarrow{OP} = \overrightarrow{r_0}$, where OP is perpendicular to the axis of rotation. In the mirror, the observed values of angular velocity, angular acceleration and position vector $\overrightarrow{O'P'}$ (image of \overrightarrow{OP}) are $\vec{\omega}$, $\vec{\alpha}$ and \vec{r} . Which of the following equations is/are correct?

A) Observed speed of P, $\vec{v} = \vec{\omega} \times \vec{r}$ B) $\vec{\omega} \times \vec{\omega_0} = 0$

C) Observed centripetal acceleration, $\vec{a_n} = -\omega^2 \vec{r}$

D)
$$\vec{r} + \vec{r_0} = 0$$

 Which of the following statements is/are correct about forces and their torques acting on an object? The object may not be in translational and/or rotational equilibrium.

A) Torque of resultant of forces acting at same point, is equal to resultant of torques of these forces, in an inertial frame of reference

B) In a non-inertial frame of reference, the point of application of pseudo force must pass through the centre of mass

C) The point of application of resultant force is the center of mass of the object, when multiple forces are acting on the object.

D) If the net force on the object is zero in an inertial frame of reference, net torque in any inertial frame of reference, about any point in space, has same magnitude and direction.

10. A rigid body is observed in equilibrium in a particular non-rotating non-inertial frame. What can you conclude, if the body is observed from an inertial frame?

A) The body is in rotational equilibrium but not in translational equilibrium

B) Net torque of all the forces on the body about its mass centre is a null vector

C) Net torque of all the forces on the body about any point that is collinear with line of the acceleration of the mass centre is a null vector

D) Net torque of all the forces on the body about all the points of a line that is parallel to the line of the acceleration of the mass centre is a null vector

11. A ladder AB is supported by a smooth vertical wall and rough horizontal floor as shown. A boy starts moving from A to B slowly. The ladder remains at rest, the pick up the correct statement(s) :



- (A) Magnitude of normal reaction by wall on ladder at point B will increase.
- (B) Magnitude of normal reaction by wall on ladder at point A will decrease.
- (C) Magnitude of normal reaction by floor on ladder at point A will remain unchanged.
- (D) Magnitude of friction force by floor on ladder at point A will remain Increase

PART-B

LEVEL-II_(APPLICATION)

SINGLE ANSWER CORRECT

1. A uniform rigid rod of mass m and length l is gently held on a smooth fixed inclined plane of angle of inclination θ , with its length perpendicular to inclined plane by an external agency as shown in figure (external agency is not shown in the figure). The angular acceleration of rod just after the external agency is removed is



2. A regular tetrahedron of side a and mass m lies on a horizontal surface of the table near the corner. A solid sphere of radius R and of same mass m lies on the one of the faces of tetrahedron along the line of symmetry with the help of a light string AB of length R connected at the vertex of the tetrahedron as shown in figure. All surfaces are smooth. The maximum value of R such that the system can be in equilibrium is (Assuming sphere does not touch the ground and table)



3. Consider a uniform thin hemispherical shell of mass M and radius R. What is it's moment of inertia about an axis passing through top most point and a periphery point of its base?



4. The moment of inertia of the uniform circular ring of mass m and radius r about an axis that passes through the centre of the ring and makes an angle 45° with the plane of the ring is

A)
$$\frac{2}{3}$$
 m r² B) $\frac{3}{4}$ m r² C) $\frac{mr^2}{2}$ D) $\frac{4}{5}$ m r²

5. A moldable blob of mater of mass M is to be situated between the planes z=0 and z=1 (see figure.), so that the moment of inertia around z-axis be as small as possible. What shape (of a sold object) should the blob take?



A) solid cylinderC) solid sphere

B) solid obtained by rotating parabola

D) solid cone

6. Which of the following setups best suits for beam balance? In all the cases axis of rotation passes through the lowest point of vertical line of center of mass and perpendicular to length



7. A uniform rod of mass M and length L (< 2R) is placed symmetrically inside a fixed hemispherical surface (see figure). An insect of mass m is moving on rod such that rod remains in equilibrium in horizontal position (x is instantaneous displacement of insect from centre of rod). Then</p>



A) situation is not possible

B) situation is possible if acceleration of insect is proportional to x

C) situation is possible if there is no friction between rod and insect

D) situation is possible if $\frac{M}{m}$ is above certain minimum value

8. Two uniform identical smooth rod AB and CD can rotate in horizontal plane about fixed vertical axes passing through 'A' and 'C' as shown. End B of rod AB collides at the midpoint of rod CD. Before collision angular speed of AB is 4 rad/s anticlockwise and CD is at rest. If angular speed of AB after collision is 1 rad/s anticlockwise the angular speed of CD after collision will be $(\theta = 53^{\circ})$



9. Rod-1 of mass M = 3kg and length L = 2m is kept on a smooth horizontal surface. Rod-2 (AB) of mass m = 3kg and length l = 1m is hinged at point A and just touches the Rod-2 as shown. Find the minimum magnitude of force F, required to move the Rod-1.



10. A trolley with rough sharp edge slides under a horizontal rod of mass m and length hinged at its end about a horizontal axis. Trolley also has a mass m and starts from rest under the action of a constant force equal to 2mg. Coefficient of friction between trolley and rod is 0.5. The velocity of the trolley when it comes to end of the rod is (initially it is at its midpoint)



11. In the figure shown, the end A of the rod of length L is being pushed down parallel to the inclined surface with a velocity = v. Let the velocity of end B = u and the angular velocity of the rod = ω . Then,



A) $u = v \cos \alpha$, upward

B) u = v, downwards D) $\omega = 2v \sin \alpha / L$

C) $\omega = v \sin \alpha / L$

12. A uniform rod of mass m and length 2l lies on smooth horizontal surface. A particle of same mass m is connected to string of length l, whose other end is connected to rod. Initially string is taut and both rod and string lies in same horizontal plane with 90° angle between them. If particle is given initially velocity v_0 perpendicular to string, then just after giving velocity v_0 to particle



- A) linear acceleration of centre of mass of rod will be $v_0^2 / 4l$
- B) angular acceleration of rod will be $6v_0^2 / 5l$
- C) tension in string will be $mv_0^2/5l$
- D) angular velocity of rod will be $v_0 / 2l$
- 13. In the figure a constant horizontal force is applied at the corner of a regular hexagon. The hexagon starts to rotate in vertical plane and does not slide during the motion. Initially the angular acceleration of the body is Γ_0 and magnitude of friction force is f_0 . After the body had rotated through 5 degree, the angular acceleration is Γ and magnitude of friction is f. Then



A) $\alpha = \alpha_0, f = f_0$ B) $\alpha > \alpha_0, f > f_0$ C) $\alpha < \alpha_0, f < f_0$ D) $\alpha > \alpha_0, f < f_0$

14. Threads of length h_1, h_2, h_3 are fastened to the vertices of a homogeneous triangular plate of weight W. The other ends of the threads are fastened to a common point as shown in the figure.(assume all the threads are taut and the plate is in equilibrium and in Horizontal plane) The **incorrect** statement among the following is



- A) Tension in the threads are proportional to their lengths
- B) Vector sum of tensions in the strings at the plate and weight of the plate is zero
- C) Tensions in the threads are inversely proportional to their lengths
- D) The centre of mass of the plate is at its centroid and it should be below the point of suspension.

ONE OR MORE THAN ONE ANSWER CORRECT

15. A ladder is constructed with three identical uniform rods each of mass m and length *l*, with their ends hinged to a light, smooth rigid ring of negligible size. The ring lies in horizontal plane. Three identical light strings, each of length $l/\sqrt{3}$ is connected to the rods as shown, between AB, BC and CA, at a distance 2l/3 from the top most point O (OA = OC = OB = 2l/3). If ladder lies on smooth horizontal surface.



A) The tension in each string is $\frac{\text{mg}}{4}$

- B) The tension in each string is $\frac{\text{mg}}{12}$
- C) The normal force exerted by ground on any rod is mg
- D) The normal force exerted by ground on any rod is $\sqrt{3}$ mg

16. In figure-I, the winch is mounted on axle, and the 6-sided nut is welded to the winch. By turning the nut with a wrench as shown in figure-II a person can rotate the winch in order to lift the block of m. Here one end of the thread is connected to block and other end is connected to periphery of winch. Which of the following statements are correct regarding lifting of block by one meter in 10 seconds.



A) By using a longer wrench, decreases the average force applied by the person on wrench

B) using of a longer wrench, reduces the work to be done by the person

C) using of a longer wrench, reduces the power to be delivered by the person

D) using of a longer wrench, reduces the number of rotations to be rotated

17. A square rigid frame ABCD is made of four uniform identical rods. Mass of frame is 'm' and side is 'a'. A particle 'P' also of mass 'm' is fixed to frame at the midpoint of rod 'AB'. Complete system is rotating about an axis perpendicular to plane of frame and passing through common centre of mass at constant angular speed ω . Tensile force at midpoint of rod 'AD' and 'BC' is T₀



A) Net force on particle 'P' = $ma\omega^2/2$

- B) Net force on frame (not including the particle) is $ma\omega^2/4$
- C) Tensile force at midpoint of rod ;CD; is $> T_0$
- D) Tensile force at midpoint of rod 'CD' is $< T_0$

18. The cable strung between two neighboring electricity posts sags a little. The mass per unit length of the cable is }, the distance between the posts is L, and the 'maximum sag' of the cable is d(d << L). Due to relatively small sag the tension in the cable can be assumed to be uniform. The cable obeys Hooke's law. Choose the correct statement(s) from the following :



A) The approximate value of tension is $\frac{L^2g}{4d}$

B) The approximate value of tension is $\frac{L^2g}{8d}$

C) The appropriate formula for calculation of elastic strain energy stored in the cable is $\frac{1}{2}x$ stress x

strain x volume

D) The appropriate formula for calculation of elastic strain energy stored in the cable is stress x strain x volume

19. A light rigid rod AB of length 3ℓ has a point mass m at end A and a point mass 2m at end B. It is kept on a smooth horizontal surface. Point C is the center of mass of the system. Initially the system is at rest. The mass 2m is suddenly given a velocity v_0 towards right. Take z axis to be perpendicular to the plane of the paper. [Points C is center of mass].



A) The minimum moment of inertia (about Z-axis), I_{zz} of the system is $5 \text{ m}\ell^2$

B) The magnitude of tension in the rod in subsequent motion is $\frac{2mv_0^2}{9\ell}$

C) The ratio of moment of inertia about Z-axis at points A and B, $\frac{I_{zz}^{A}}{I_{zz}^{B}} = 2$

D) Point C remains stationary during subsequent motion

20. A light rigid ring of radius R = 1m has two identical beads A & B each of mass m = 1kg. The ring is hinged at its top point O in such a way that it can rotate in the vertical plane freely about a horizontal axis into the plane of the diagram shown. Bead A is rigidly attached to ring and bead B is free to slide along the ring without friction. Immediately after the bead B is released



- A) The acceleration of bead B has a magnitude of $\left(\sqrt{10}\right)\left(\frac{g}{3}\right)m/s^2$
- B) The ratio of magnitudes of hinge reaction at the point O along x and y axis is equal to 1
- C) The acceleration of bead A is $\left(\sqrt{2}\right)\left(\frac{g}{3}\right)m/s^2$
- D) The net reaction force from the hinge O is $\left(\sqrt{2}\right)\left(\frac{20}{3}\right)N$
- 21. A uniform rod with length l is attached with a weightless thread (whose length is also l) to the ceiling at point A. The bottom end of the rod rests on a frictionless horizontal floor at point B, which is exactly below point A. The length of AB is H, l < H < 2l. The rod begins to slide from rest. The maximum velocity of the centre of rod during subsequent motion



D) Cannot be determined

22. Two identical uniform solid cone A and B can rotate freely about their fixed axis which is also central axis of the two cones as shown in the figure. The cone A is rotated with a constant angular velocity $\check{S}_A = \check{S}_0$, it is given that there exist friction between the two cones. Assume normal reaction is uniformly distributed between the two cones



Choose the correct option(s)

- A) Final angular velocity of the cone B $\check{S}_{B} = \check{S}_{0}$
- B) Net work done by the friction force on cone B $\check{S}_{f_r}\Big|_B \Rightarrow \frac{3(3-2\sqrt{2})}{20} MR^2 \check{S}_0^2$
- C) Final angular velocity of the cone B $\breve{S}_{B} = (\sqrt{2} 1)\breve{S}_{0}$
- D) Net work done by the friction force on cone B $\check{S}_{f_r}\Big|_B \Rightarrow \frac{3}{20} MR^2 \check{S}_0^2$
- 23. Consider a fixed smooth hemispherical bowl of radius R with its opening horizontal. One end of a light rod of length $\ell = \sqrt{3}$ R is hinged at a point A on the brim of the bowl. A small bead of mass m is attached at the other end of the rod. Initially the bead is held at a point B also on the brim and then released. Acceleration due to gravity at the place is g. Mark the **CORRECT** option(s) at the instant when the bead reaches its lowest position.



- a) Longitudinal stress in the rod is tensile in nature.
- b) Tension in the rod is of magnitude 3 mg
- c) speed of the bead is $\sqrt{2gR}$
- d) Acceleration of the bead is 2g

COMPREHENSION TYPE

PASSAGE

A uniform heavy square plate ABCD is connected to four identical suspension springs (which obey Hooke's law) as shown and is initially at rest in equilibrium on horizontal floor. This system is shown in figure – 1. There is a step of vertical height 8cm as shown in figure-2. The plate spring system is now rested on the step as shown (partial diagram from the front on view) in figure – 3, such that springs connected to B, C, D rest on horizontal floor and only spring connected to A rests on the step. Assume all springs stay vertical. The plate is again left to achieve static equilibrium.



24. The vertical displacement of vertex A of the plate from figure-1 equilibrium state to figure-3 equilibrium state is closest to

A) 8cm upward B) 6cm upward C) 4cm upward D) 2cm upward

25. The vertical displacement of vertex B of the plate from figure-1 equilibrium state to figure-3 equilibrium state is closest to

A) 4cm upward B) 2cm downward C) 2cm upward D) zero

26. The vertical displacement of vertex C of the plate from figure-1 equilibrium state to figure-3 equilibrium state is closest to

A) 4cm downward B) 2cm upward C) 2cm downward D) 1cm downward

PASSAGE

A uniform rod of mass M and length L is placed freely on a rough horizontal table having friction coefficient μ . A horizontal force F is applied perpendicular to the rod at one of its ends. The force F is increased gradually from zero and it is observed that when its value becomes F_0 , the rod just begins to rotate about point O



INTEGER TYPE

29. Four identical uniform rods are connected in series and hanging from the ceiling. Assume joints to be perfectly smooth. When the system shown above is in equilibrium, what is $\frac{\tan \alpha}{\tan \beta}$?



30. A thin uniform rod *AB* of mass m = 1 kg moves translationally with acceleration $a = 2 \text{ m/s}^2$ due to two anti parallel forces F_1 and F_2 . The distance between the points at which these forces are applied is equal to l = 20 cm. Besides, it is known that $F_2 = 5$ N. Find the length of the rod.



31. Figure shows a paper role mounted on horizontal fixed axis. Paper is drawn at a constant speed V. The thickness of paper is b and r is the instantaneous radius of the roll (b << r such that the time taken to complete one rotation can be taken as $\frac{2\pi r}{v}$ approximately). The angular acceleration of the

paper roll (approximately) is given by $\alpha = \frac{bV^a}{2\pi r^c}$. Then $a + c = _$



32. A horizontally uniform rod AB of mass m and length l_0 rotates freely about a stationary vertical axis OO' passing through the end A, which is located exactly midway between O and O' and OO' = l. At what angular velocity (in units) of the rod is the horizontal component of the force acting on the lower end of the axis OO' equal to zero. (g = 10 m/s², l = 5m, $l_0 = 3m$)



33. A uniform rod rests against a smooth vertical wall and a rough horizontal surface. A horizontal force F is applied at the centre of mass of the rod such that the rod is in static equilibrium and the resultant of F and the weight of the rod intersect at the point P shown in the figure. The angle (in degree) of friction for friction force between the rod and horizontal surface is $(10x)^0$. Find *x* value



34. There is a thin long rectangular strip ABCD, as shown. Corner A is joined to corner D and corner B is joined to corner C to form closed twisted ring (called Mobius Strip). An ant starts from some point 'P' on the strip, moving along the circumference with constant speed, and come back to P after time T_1 . If, corner A is joined to corner C and corner B is joined to correr D, same ant with same speed along circumference takes time T_2 to return to P. Find T_1/T_2 .



35. A uniform rod has mass m = 2kg and length l = 13m. One end of the rod is pulled with a constant velocity of $v_0 = 34 \text{ m/s}$ along a frictionless horizontal floor in the negative x direction. The other end is moving along a parabolic fixed curve. The equation of the parabola is $x^2 = 20y$. Find the angular velocity of the rod (in rad/s) when the end point 'B' is at (10, 5)



36. A particle is moving along a circle of radius R, particle is moving under the influence of an unknown gravitation central force F which is always pointing towards point O. Particle is projected from point P which is directly opposite of O with a initial velocity V_0 as shown in the figure. If the expression

of force F is given by $F = \frac{32m^x \cdot R^y \cdot V_0^z}{r^k}$, where x, y, z and k are integers. Then the value of

 $\left\{ \left(x+y+z\right) -k\right\}$ will be ?



37. A thin rod of mass m and length 2ℓ is placed horizontally and perpendicular to a horizontal rough nail, as shown in figure and set free. The point of contact of the rod with the nail is $\ell/3$ distance away from the centre of rod. If the rod starts slipping when it forms an angle θ with the horizontal

and the coefficient of friction of rod with nail is μ , then find $\frac{\mu}{\tan \theta}$.

PART-A_LEVEL-I_(THEORY)_KEY

1	2	3	4	5	6	7	8	9	10	11
3	D	C	В	А	В	D	ABC	AD	ABD	ACD

PART-B_LEVEL-II_(APPLICATION)_KEY

1	2	3	4	5	6	7	8	9	10
D	А	А	В	А	D	В	А	4	В
11	12	13	14	15	16	17	18	19	20
D	С	D	С	AC	А	BD	BC	BC	ABCD
21	22	23	24	25	26	27	28	29	30
В	BC	BD	В	С	С	С	В	3	1 m
31	32	33	34	35	36	37			
5	2	3	2	2	2	2			